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Can light Goldstone boson loops counter the 'S-argument' against Technicolor?

Sean Fleming

Department of Physics and Astronomy
Northwestern University, Evanston, IL 60208

Ivan Maksymyk

Theory Group, Department of Physics
University of Texas, Austin, TX 78712

Abstract

We examine the oblique correction phenomenology of one-family Technicolor with light pseudo-Goldstone bosons. From loop calculations based on a gauged chiral lagrangian for Technicolor, we are lead to conclude that even though loops with light Goldstone bosons give a negative contribution to S measured at the Z-pole, this effect is not sufficiently large to unambiguously counter the 'S-argument' against one-family Technicolor. This result cannot be guessed *a priori*, but must be explicitly calculated. Our analysis entails an extended version of the STU oblique parametrization of Peskin and Takeuchi. In principle, this extended formalism ($STUVWX$) must be used when there are light new particles in loops.

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1. Introduction

The precision e^+e^- collision data currently being collected will allow for a real probing of electroweak radiative corrections and physics beyond the Standard Model. One method for parametrizing such effects is the STU oblique formalism of Peskin and Takeuchi [1], which can be used to encode the effects of new physics electroweak gauge-boson self-energies when these self-energies can be effectively expressed as linear functions of q^2,

$$\Pi(q^2) = \Lambda^2 \left(a_0 + a_1 \frac{q^2}{\Lambda^2} + O\left(\frac{q^4}{\Lambda^4}\right)\right) \tag{1}$$

with a_0 and a_1 some constants. The STU approximation is valid when the new physics scale Λ is much greater than the scale at which experiments are performed, *i.e.* the Z-pole and below. The set of three parameters S, T and U has recently been extended, in [2] [3] , to the case of light new physics, for which the self-energies would *a priori* be some general, complicated functions of q^2. The extended version of the formalism involves the six parameters S, T, U, V, W and X. In principle, the extended version must be used if loop contributions to the oblique corrections entail light new particles with masses in the range $\sim M_Z$ or less. The essence of the $STUVWX$ formalism is that the total theoretical expression for any of the precision electroweak observables measured at $q^2 \approx 0$, $q^2 = M_Z^2$ or $q^2 = M_W^2$ can be expressed as a standard model prediction plus some linear combination of S through X. It turns out, moreover, that all Z-pole observables can be expressed in terms of only two parameters, S' and T', which are linear combinations of S through X.

It is of interest to apply oblique correction formalisms to models of dynamical symmetry breaking such as Technicolor [4], as this type of new physics couples most strongly to gauge bosons and therefore essentially generates oblique effects. It is well known that, only a few years ago, oblique correction considerations hinging on the parameter S tended to rule out certain models of Technicolor [5] [6]. Least mean square fits involving the three parameters S, T, and U suggest that the measured value of S is consistent with zero, or even slightly negative, while theoretical calculations determined S to be large and positive. For example the logarithmically divergent part of the one loop chiral lagrangian contibution to S is typically positive in Technicolor theories. In addition to this "low -energy" piece there is a "high-energy" contribution which, when calculated by scaling the parameters of the QCD chiral lagrangian, is also positive.

The S-argument against Technicolor was countered in [6], where it was pointed out that the high-energy contribution determined from scaling the parameters of the QCD chiral lagrangian represents an upper bound, and that other methods used to estimate

this contribution result in a smaller or negative value for the high-energy piece. The authors of [6] naïvely estimate the high-energy contribution by calculating the one loop technifermion diagrams, and find that, after adding it to the low-energy piece, the S-argument against Technicolor can be invalidated. Thus, ref. [6], entitled "Revenge of the one-family Technicolor models," re-established the possible phenomenological viability of this model.

The calculations in the present article were embarked upon in hope of further legitimizing Technicolor. Our point of departure was the idea that, strictly speaking, the results of a fit of the three parameter set STU to experimental data can only be applied when the physical Goldstone bosons in Technicolor are thought to be heavy. We therefore set out to explore the possibility that some of them are light (but just heavy enough to have so far escaped direct detection), and to determine whether, in such a scenario, the theoretical values of the new parameters V, W and X can be as large as various estimates of S. If so, the parameter $S' = S + 4(c^2 - s^2)X + 4c^2s^2V$ observed at the Z-pole might be consistent with experiment. Then one might say that the VWX-argument undoes the original S-argument against Technicolor.

This article is organized as follows. In Section 2, we review how the STU formalism can be extended to the case of light new physics. The extended formalism entails the six parameters $STUVWX$. In Section 3, we review the gauged chiral lagrangian (which is an effective lagrangian for Technicolor) and calculate the one-loop oblique corrections, paying close attention to the sign of S', and to the ramifications of loops involving light Goldstone bosons. We conclude in Section 4.

2. $STUVWX$ Formalism

2.1) Extending the STU Parameter Set

The STU formalism of Peskin and Takeuchi [1] provides an elegant means of parametrizing new physics effects on electroweak observables, when the new physics couples most strongly to gauge bosons (*i.e.* oblique corrections). This formalism allows us to write a wide range of observables as a standard model prediction plus some linear combination of the three parameters S, T and U. The STU parametrization is based explicitly on the assumption that new physics is heavy, and that new physics contributions to gauge-boson self-energies are therefore linear functions of q^2, *i.e.* of the form of eq. (1).

If the heavy new physics assumption is dropped, the gauge-boson self-energies have some complicated dependence on q^2 that cannot be adequately expressed using the first

few terms of a Taylor expansion. Nonetheless, since precision observables are associated only with the scales $q^2 \approx 0$, $q^2 = M_Z^2$ or $q^2 = M_W^2$, it turns out that it is possible in practice to parametrize oblique effects due to light new physics in terms of only six parameters S, T, U, V, W and X. These are defined as [2] [3]

$$\begin{aligned}
\alpha S =& -4s^2c^2\widehat{\Pi}_\gamma(0) + \frac{4s^2c^2}{M_Z^2}\big(\Pi_Z(M_Z^2) - \Pi_Z(0)\big) \\
& -4(c^2-s^2)sc\,\widehat{\Pi}_{Z\gamma}(0) && (2)\\
\alpha T =& \frac{\Pi_W(0)}{M_W^2} - \frac{\Pi_Z(0)}{M_Z^2} && (3)\\
\alpha U =& -4s^4\widehat{\Pi}_\gamma(0) + \frac{4s^2}{M_W^2}\big(\Pi_W(M_W^2) - \Pi_W(0)\big) \\
& -\frac{4s^2c^2}{M_Z^2}\big(\Pi_Z(M_Z^2) - \Pi_Z(0)\big) - 8cs^3\widehat{\Pi}_{Z\gamma}(0) && (4)\\
\alpha V =& \Pi_Z'(M_Z^2) - \left[\frac{\Pi_Z(M_Z^2) - \Pi_Z(0)}{M_Z^2}\right] && (5)\\
\alpha W =& \Pi_W'(M_W^2) - \left[\frac{\Pi_W(M_W^2) - \Pi_W(0)}{M_W^2}\right] && (6)\\
\alpha X =& -sc\left[\widehat{\Pi}_{Z\gamma}(M_Z^2) - \widehat{\Pi}_{Z\gamma}(0)\right] && (7)
\end{aligned}$$

where $\widehat{\Pi}(q^2) \equiv \Pi(q^2)/q^2$, and where $\Pi'(q^2)$ denotes the ordinary derivative with respect to q^2. The V, W and X are intentionally defined so that they vanish when the self-energies are linear functions of q^2 only, in which case the STU parametrization is exactly recovered.

We now illustrate how the above parameters appear in expressions for observables. First consider the low-energy neutral current asymmetries, which depend only on an effective $\sin^2\Theta_W$ evaluated at $q^2 \approx 0$. Just as in the Peskin-Takeuchi parametrization, this quantity is given by

$$s^2(0)_{\text{eff}} = s^2(0)_{\text{eff}}^{SM} + \frac{\alpha S}{4(c^2-s^2)} - \frac{c^2s^2\ \alpha T}{c^2-s^2} \qquad (8)$$

where $s^2(0)_{\text{eff}}^{SM}$ is the standard model prediction for some given asymmetry, and where the particular linear combination of S and T is common to all asymmetries measured at $q^2 \approx 0$.

However, as to the Z-pole neutral current asymmetries such as A_{LR} and A_{FB}, the oblique corrections to the effective $\sin^2\Theta_W$ at the Z-pole are given by

$$s^2(M_Z^2)_{\text{eff}} = s^2(M_Z^2)_{\text{eff}}^{SM} + \frac{\alpha S}{4(c^2-s^2)} - \frac{c^2s^2\ \alpha T}{c^2-s^2} + \alpha X. \qquad (9)$$

Here, the parameter X represents a supplementary Z-pole effect, defined in eq. (7).

In the full $STUVWX$ formalism, the neutral current vertex at the Z-pole is multiplied by an overall oblique correction factor $(1 + \alpha T + \alpha V)$. Thus, for example, the width of Z-decay to neutrinos is given by

$$\Gamma(Z \to \nu\overline{\nu}) = \Gamma(Z \to \nu\overline{\nu})^{SM}(1 + \alpha T + \alpha V). \tag{10}$$

In studying eqs. (8), (9) and (10), one sees that when one drops the assumption of heavy new physics and, with it, the corresponding linear approximation, it is a simple matter to systematically incorporate the new physics oblique effects into expressions for Z-pole observables.

Similarly, the width of W-decay to a single lepton-neutrino pair is given by

$$\Gamma(W \to l\overline{\nu}) = \Gamma(W \to l\overline{\nu})^{SM} \left(1 - \frac{\alpha S}{2(c^2 - s^2)} + \frac{c^2\ \alpha T}{(c^2 - s^2)} + \frac{\alpha U}{4s^2} + \alpha W\right). \tag{11}$$

Finally, note that in the $STUVWX$ formalism, the mass of the W-boson is given by a formula identical to that arising in the STU formalism, namely

$$M_W^2 = (M_W^2)^{SM} \left(1 - \frac{\alpha S}{2(c^2 - s^2)} + \frac{\alpha c^2 T}{c^2 - s^2} + \frac{\alpha U}{4s^2}\right). \tag{12}$$

2.2) Oblique Parameters for Z-pole Measurements

The formalism described above is the most natural extension of the STU parameterization, though it does have the disadvantage that X and V appear in the expressions for Z-pole observables. It is, however, possible to cast the oblique corrections to all Z-pole observables in terms of only two parameters, which, following [7], we may conveniently define as

$$\begin{aligned} S' &= S + 4(c^2 - s^2)X + 4c^2s^2V \\ T' &= T + V. \end{aligned} \tag{13}$$

The effective vertex for neutral currents at the Z-pole is now given by

$$i\Lambda^\mu_{\rm nc}(q^2 = M_Z^2) = -i\,\frac{e}{sc}\,(1 + \frac{1}{2}\alpha T')\,\gamma^\mu \left[I_3^f\gamma_L - Q^f\left(s^2 + \frac{\alpha S'}{4(c^2 - s^2)} - \frac{c^2s^2\ \alpha T'}{c^2 - s^2}\right)\right]. \tag{14}$$

So, in confronting some model of light new physics with Z-pole data, one would calculate S' and T' rather than S and T. The ϵ parameters of Altarelli and Barbieri [11] are connected to these parameters by $\epsilon_1 = \alpha T'$ and $\epsilon_3 = \alpha S'/(4s^2)$. With S' and T' defined this way, the low-energy neutral-current observables now depend on S', T', V, and X; the W-mass depends on S', T', U, V, and X.

The results of fits to precision data for the parameters STU can be found in [9], and for $STUVWX$ in [3]. Fits to the most recent LEP and SLC data (Winter 1995) are presented in [10], the result being

$$\begin{aligned} S' &= -0.20 \pm 0.20 \\ T' &= -0.13 \pm 0.22 \\ \alpha_{\rm s}(M_Z) &= 0.127 \pm 0.005 \end{aligned} \tag{15}$$

3. Calculation of S through X in One-Family Technicolor

3.1) Gauged Chiral Lagrangian for Technicolor

Our approach consists of using an effective lagrangian (the gauged chiral lagrangian [12] [13] [14] [15] [16]) to calculate one-loop contributions to the self-energies of the electroweak gauge bosons.

Let us consider the “one-family” model of Technicolor. In this model, a chiral symmetry $SU(8)_L \times SU(8)_R$ is realized on a set of technifermions of eight flavours: (U^α_r, D^α_r, U^α_b, D^α_b, U^α_g, D^α_g, E^α, N^α). There is a flavour of technifermion for each distinguishable member of a one-family representation of the usual gauge group $G \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$. ‘$\alpha$’ indexes Technicolor, a new color-like force. The technifermions have the same quantum numbers under G as the corresponding ordinary fermions. It is assumed that ordinary fermions are singlets under Technicolor. The new color-like Technicolor force becomes strong at some scale Λ_{TC} in the TeV range, resulting in the breaking of the chiral symmetry to $SU(8)_V$ and in the formation of Goldstone bosons, called “technipions,” which are bound states of two technifermions. This is exactly analogous to the formation of pions and the breaking of chiral symmetry in ordinary hadronic physics.

Following [13] [15] and [16], we define

$$U = \exp \frac{i 2 X_i \Pi_i}{v} \tag{16}$$

where Π_i are the 63 technipion fields associated with the breaking of the chiral symmetry and where X_i are the 63 8×8 traceless hermitian matrices that generate $SU(8)$, normalized so that

$$Tr\,[X_i X_j] = \frac{1}{2}\,\delta_{ij}. \tag{17}$$

The gauged chiral lagrangian is written as

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}' \tag{18}$$

where the most important terms are found in

$$\mathcal{L}_{\rm kin} = \frac{v^2}{4} Tr\left[(D^\mu U)^\dagger\; D_\mu U\right], \tag{19}$$

and where $\mathcal{L}'$, contains a set of $SU_L(2) \times U_Y(1)$-invariant terms, including terms up to some given order in derivatives. The $SU_L(2) \times U_Y(1)$ covariant derivative is given by

$$D^\mu U = \partial^\mu U - ig\sqrt{N_d} W_i^\mu T_i\, U + ig'\sqrt{N_d}(UT_3 B^\mu - [\mathcal{Y}, U]\, B^\mu) \tag{20}$$

where N_d is the number of technidoublets, where

$$T_i = \frac{1}{\sqrt{N_d}}\,\tau_i \otimes I_{N_d}, \tag{21}$$

and where, for one-family Technicolor (with $N_d = 4$), we have

$$\mathcal{Y} = \frac{1}{2}\begin{pmatrix} \frac{1}{3}\tau_0 & & & \\ & \frac{1}{3}\tau_0 & & \\ & & \frac{1}{3}\tau_0 & \\ & & & -\tau_0 \end{pmatrix}. \tag{22}$$

We define $\tau_0 \equiv \frac{1}{2} I_2$ and $\tau_i \equiv \frac{1}{2}\sigma_i$. The explicit $\sqrt{N_d}$ displayed in eq. (20) assures that, for example, the mass of the W-boson works out to $M_W^2 = \frac{1}{4} g^2 N_d v^2$ as required. The gauged chiral lagrangian is invariant under the local $SU_L(2) \times U_Y(1)$ transformation in which the Goldstone bosons transform according to

$$U \rightarrow\; e^{i(\beta\mathcal{Y} + \alpha_i T_i)}\; U\; e^{-i\beta(\mathcal{Y} + T_3)}, \tag{23}$$

and in which the gauge bosons transform according to the usual Yang-Mills transformation rule. Of the 63 Goldstone bosons, three are eaten, leaving 60 physical pseudo-scalars in the theory.

3.2) Calculation of "High-Energy" Contribution by Scaling of QCD Results

Before proceding with our loop calculations, we will look at the sector $\mathcal{L}'$ appearing in eq. (18). This sector consists of an expansion in derivatives of all the locally $SU_L(2)\times U_Y(1)$-invariant terms that one can construct from gauge-boson and Goldstone boson fields. Among the interactions included in $\mathcal{L}'$ is the operator $B_{\mu\nu}W_i^{\mu\nu}Tr\left[U^\dagger T_3 U T_i\right]$, which gives a "high-energy" contribution to the oblique parameter S. The operator's coefficient is defined via,

$$\mathcal{L}_{\rm eff}^{QCD} = L_{10}^{QCD} g g' B_{\mu\nu} W_i^{\mu\nu} Tr\left[U^\dagger \tau_3 U \tau_i\right] + \ldots, \tag{24}$$

where the experimental value

$$L_{10}^{QCD}(\Lambda_{QCD}) = -5.4 \pm 0.3 \times 10^{-3} \tag{25}$$

is determined from measurments of the pion charge and the decay $\pi \to e\nu\gamma$ [18]. To find the correct normalization of this operator in our conventions for technicolor with N_d doublets, we note that the contribution to a gauge-boson two-point function is $L_{10}^{QCD} N_d N_{TC}/N_{QCD}$. This direct physical association requires that we write

$$\mathcal{L}_{\rm eff}^{TC} = N_d \frac{N_{TC}}{N_{QCD}} L_{10}^{QCD} g g' B_{\mu\nu} W_i^{\mu\nu} Tr\left[U^\dagger T_3 U T_i\right] + \ldots . \tag{26}$$

The gauge boson two-point function embedded in the above equation is

$$\mathcal{L}_{\rm eff}^{TC} = \frac{N_d}{2}\frac{N_{TC}}{N_{QCD}} L_{10}^{QCD} g g' B_{\mu\nu} W_3^{\mu\nu} + \ldots . \tag{27}$$

Since S is generically associated with $-32\pi sc/e^2$ times the coefficient of the $B_{\mu\nu}W_3^{\mu\nu}$ term, we have

$$S(\Lambda_{TC}) = -16\pi \frac{N_d N_{TC}}{N_{QCD}} L_{10}^{QCD}(\Lambda_{QCD}) \sim +1. \tag{28}$$

It is this large positive "high-energy" contribution, combined with the positive logarithm that is calculated in the next subsection, that, at the outset, renders the model unviable.

(See eq. (15).) In ref. [6], however, it was pointed out that the high-energy contribution can be estimated naïvely by simply calculating the technifermion loops, yielding a result that can be as low as -0.2.

Finally, the entire phenomenological value of the measured quantity $S(M_Z)$ is given by

$$S(M_Z) = S(\Lambda_{TC}) + S \tag{29}$$

where S henceforth refers to the the contribution obtained by calculating gauge-boson self-energies involving physical Goldstone boson loops. Such loops are calculated in the next subsection. The logarithmically divergent parts of S give the renormalization group scaling of $S(\mu)$ from Λ_{TC} down to M_Z.

3.3) Goldstone Boson Loop Calculations

The interactions pertinent to our one-loop calculations are the Goldstone-Goldstone-gauge-boson (GGg) and Goldstone-Goldstone-gauge-gauge-boson ($GGgg$) interactions embedded in eq. (19). The relevant Feynman rules are given in Fig. 1. Such couplings contribute to the gauge boson self-energies through the one-loop diagrams shown in Fig. 2.

The one-loop contributions to oblique corrections in the gauged chiral lagrangian have been studied in [14] [16] [19]. In refs. [14] and [16], the *(logarithmically) divergent* parts of various electroweak observables were calculated only. Since the divergent parts of the self-energies turn out to be linear functions of q^2, these analyses fit into the framework of the STU formalism.

The author of ref. [19], on the other hand, explicitly considered the possibility of light new particles, and thus adopted the $STUVWX$ formalism. In performing one-loop calculations with a degenerate triplet of Goldstone bosons, this author was concerned only with the finite parts of the gauge-boson self-energies, and as a result, did not display the divergent parts (all of which all reside in the parameter S). Moreover, in ref. [19], it was not asked whether the VWX-argument could help undo the S-argument against Technicolor.

• Goldstone Boson Isotriplets

Calculating the loop contributions from a degenerate non-self-conjugate isotriplet of Goldstone bosons (and its conjugate triplet), we obtain the following self-energy pieces:

$$\Pi_\gamma(q^2) = e^2(2+3y^2)(I(q^2) - 2J)$$

$$
\begin{aligned}
\Pi_{Z\gamma}(q^2) &= e^2\frac{(1-2s^2-3s^2y^2)}{sc}(I(q^2)-2J) \\
\Pi_Z(q^2) &= e^2\left(\frac{(1-2s^2)^2+6s^4y^2)}{2s^2c^2}I(q^2)+\frac{2}{c^2}(2c^2-3s^2y^2)J\right) \\
\Pi_W(q^2) &= \frac{e^2}{2s^2}I(q^2)
\end{aligned}
\tag{30}
$$

where y is the hypercharge of the triplet, defined through $Q = I_3 + Y$, and where $I(q^2)$ and J correspond to the contributions from from figures 2a and 2b respectively. They are defined as

$$
I(q^2) = \frac{1}{8\pi^2}\left[\left(m_\pi^2 - \frac{q^2}{6}\right)\left(\frac{1}{\epsilon'}+\log\frac{\mu^2}{m_\pi^2}\right) - \int_0^1 dx(m_\pi^2 - q^2(x-x^2))\log\left(1-\frac{q^2}{m_\pi^2}(x-x^2)\right)\right] \tag{31}
$$

$$
J = \frac{1}{16\pi^2}m_\pi^2\left[\frac{1}{\epsilon'}+\log\frac{\mu^2}{m_\pi^2}\right] \tag{32}
$$

where $1/\epsilon' \equiv 2/(n-4) - \gamma + 1 + \log 4\pi$. We interpret the $1/\epsilon'$ coefficient as determining the logarithmic scaling of S from Λ_{TC} down to M_Z.

Using the definitions of $S{-}X$ given in eqs. (2) through (7), we obtain for the degenerate non-self-conjugate isotriplet and its conjugate:

$$
\begin{aligned}
\alpha S &= \frac{e^2}{24\pi^2}\log\frac{\Lambda_{TC}^2}{M_Z^2} + \text{convergent pieces} \\
\alpha T &= 0 \\
\alpha U\frac{4\pi^2}{e^2} &= -\frac{2s^2c^2}{3} + s^4y^2 - \int_0^1 dx\left(\frac{m_\pi^2}{M_W^2}-(x-x^2)\right)\log\left(1-\frac{M_W^2}{m_\pi^2}(x-x^2)\right) \\
&\quad + ((1-2s^2)^2+6s^4y^2)\int_0^1 dx\left(\frac{m_\pi^2}{M_Z^2}-(x-x^2)\right)\log\left(1-\frac{M_Z^2}{m_\pi^2}(x-x^2)\right) \\
\alpha V &= \frac{e^2}{16\pi^2s^2c^2}((1-2s^2)^2+6s^4y^2)\left[\frac{m_\pi^2}{M_Z^2}\int_0^1 dx\log\left(1-\frac{M_Z^2}{m_\pi^2}(x-x^2)\right)+\frac{1}{6}\right] \\
\alpha W &= \frac{e^2}{16\pi^2s^2}\left[\frac{m_\pi^2}{M_W^2}\int_0^1 dx\log\left(1-\frac{M_W^2}{m_\pi^2}(x-x^2)\right)+\frac{1}{6}\right] \\
\alpha X &= \frac{e^2}{8\pi^2}(1-2s^2-3s^2y^2)\left[\int_0^1 dx\left(\frac{m_\pi^2}{M_Z^2}-(x-x^2)\right)\log\left(1-\frac{M_Z^2}{m_\pi^2}(x-x^2)\right)+\frac{1}{6}\right].
\end{aligned}
\tag{33}
$$

(The results for a degenerate self-conjugate isotriplet can be obtained from the above expressions by setting $y{=}0$ and dividing by two.)

Note that the logarithmic divergence in S is positive, and, as it turns out, strictly independent of the hypercharge y. Thus, no exotic values of hypercharge can be evoked to render S negative. T is exactly zero (because of the degeneracy of the triplet). U, V, W and X are finite, and therefore can be evaluated unambiguously. Below, we display the results for V and X in two interesting limits: $m_\pi = \frac{1}{2}M_Z$ (for which an exact expression is easily obtained) and $m_\pi \gg M_Z$ (for which we can expand in M_Z^2/m_π^2). The results are given, respectively, by

$$\begin{aligned}
\alpha V &= \frac{e^2}{16\pi^2 s^2 c^2}\left((1-2s^2)^2+6s^4y^2\right)\left(\quad -\ \frac{1}{3} \quad , \quad -\ \frac{1}{60}\frac{M_Z^2}{m_\pi^2} \quad + \quad O(\frac{M_Z^4}{m_\pi^4}) \quad \right) \\
\alpha X &= \frac{e^2}{8\pi^2}\left(1-2s^2-3s^2y^2\right)\left(\quad +\ \frac{1}{9} \quad , \quad +\ \frac{1}{60}\frac{M_Z^2}{m_\pi^2} \quad + \quad O(\frac{M_Z^4}{m_\pi^4}) \quad \right).
\end{aligned} \tag{34}$$

The above results for the large m_π limit have the peculiar feature that the coefficient of the first term in the Taylor expansion is surprisingly small. Thus we see that as m_π increases from $M_Z/2$ to, say, $2M_Z$, the size of V or X is diminished by at least one full order of magnitude!

- Goldstone Boson Isosinglets

The contributions to the self-energies due to a non-self-conjugate singlet are given by

$$\begin{aligned}
\Pi_\gamma(q^2) &= e^2y^2(I(q^2)-2J) \\
\Pi_{Z\gamma}(q^2) &= -e^2y^2\frac{s}{c}(I(q^2)-2J) \\
\Pi_Z(q^2) &= e^2y^2\frac{s^2}{c^2}\left(I(q^2)-2J\right) \\
\Pi_W(q^2) &= 0.
\end{aligned} \tag{35}$$

With these self-energy contributions, we obtain the following results for the parameters S through X:

$$\begin{aligned}
\alpha S &= -\frac{e^2s^4y^2}{2\pi^2}\left[\int_0^1 dx\left(\frac{m_\pi^2}{M_Z^2}-(x-x^2)\right)\log\left(1-\frac{M_Z^2}{m_\pi^2}(x-x^2)\right)+\frac{1}{6}\right] \\
\alpha T &= 0 \\
\alpha U &= \frac{e^2s^4y^2}{2\pi^2}\left[\int_0^1 dx\left(\frac{m_\pi^2}{M_Z^2}-(x-x^2)\right)\log\left(1-\frac{M_Z^2}{m_\pi^2}(x-x^2)\right)+\frac{1}{6}\right] \\
\alpha V &= \frac{e^2s^2y^2}{8c^2\pi^2}\left[\frac{m_\pi^2}{M_Z^2}\int_0^1 dx\log\left(1-\frac{M_Z^2}{m_\pi^2}(x-x^2)\right)+\frac{1}{6}\right] \\
\alpha W &= 0 \\
\alpha X &= -\frac{e^2s^2y^2}{8\pi^2}\left[\int_0^1 dx\left(\frac{m_\pi^2}{M_Z^2}-(x-x^2)\right)\log\left(1-\frac{M_Z^2}{m_\pi^2}(x-x^2)\right)+\frac{1}{6}\right].
\end{aligned} \tag{36}$$

(To obtain the result for a self-conjugate singlet, one simply sets y to zero, *i.e.* there is no contribution from a self-conjugate singlet.) The above formulae illustrate that the contributions due to a non-self-conjugate singlet are all finite. Interestingly, one discovers, upon evaluation of the integral, that the above result for S is generally negative. For $m_\pi = M_Z/2$, we have

$$\alpha S = -\frac{e^2 s^4 y^2}{2\pi^2}\left(\frac{1}{9}\right), \tag{37}$$

and for $m_\pi \gg M_Z$, we have

$$\alpha S = -\frac{e^2 s^4 y^2}{2\pi^2}\left(\frac{1}{60}\frac{M_Z^2}{m_\pi^2}\right). \tag{38}$$

This negative value could be taken as a reassuring sign if one wanted to further establish the phenomenological feasibility of Technicolor. However, it must be appreciated that of the 60 physical Goldstone bosons in one-family Technicolor, only three pairs of particles (the coloured isosinglets designated as T_c and $\overline{T}_c$ in [4]) are non-self-conjugate singlets. The great majority of the Goldstone bosons are arranged in triplets, and therefore the negative S contributions from the few non-self-conjugate singlets cannot effectively counter the positive contributions from the many triplets.

3.4) Numerical Estimates

Estimates for the masses of the various Goldstone bosons are presented in [4]. Most of these particles (those designated as T_c^i, $\overline{T}_c^i$ and θ_a^i, constituting a total of 14 triplets) are expected to have masses of roughly $m_\pi = 200$ GeV. Taking this value for m_π and taking $\Lambda_{TC} \approx 1$ Tev, one obtains for an individual (self-conjugate) triplet

$$\begin{aligned} S &= \frac{1}{12\pi}\log\frac{\Lambda_{TC}^2}{M_Z^2} + \text{convergent pieces} \quad \sim \quad O(0.1) \\ U, V, W, X &\sim O(0.0001). \end{aligned} \tag{39}$$

The essential result is therefore that, for a triplet of mass $m_\pi = 200$ GeV, the chiral loop contribution to S is significantly larger than the contribution to the other parameters.

Let us next examine the case of lighter Goldstone bosons. In one-family Technicolor, there does exist one (self-conjugate) triplet of particularly light physical Goldstone bosons, the P_i, with mass estimated to be less than 100 GeV [4]. To estimate the most dramatic possible contribution of this triplet, let us assume (as in eq. (34)) that $m_\pi = M_Z/2$, *i.e.*

that the technipions are as light as possible while being just out of reach of direct detection. In this case, evaluation of eqs. (33) and (34) gives

$$
\begin{aligned}
S &= \frac{1}{12\pi}\log\frac{\Lambda_{TC^2}}{M_Z^2} + \text{convergent parts} \quad \sim \quad O(0.1) \\
T &= 0 \\
U &= -\,0.006 \\
V &= -\,0.02 \\
W &= -\,0.02 \\
X &= +\,0.005.
\end{aligned} \tag{40}
$$

From eq. (40), it can be appreciated that the oblique quantity which is measured at the Z-pole, $S' \equiv S + 4(c^2 - s^2)X + 4c^2s^2V$, does not receive an appreciable negative contribution from the V and X terms. Therefore, it appears that the VWX-argument does not help to undo the S-argument against one-family Technicolor. This result cannot be guessed *a priori*, but must be determined through explicit calculation. Surprises and "conspiracies" can occur in these calculations. For example, it has been noticed [8] that in extensions of the Standard Model involving doublets of fermions or multiplets of scalar bosons, the photon-Z self-energy is proportional to the very small quantity $\frac{1}{4} - s^2$, so that X is by chance much smaller than the other parameters; in the present calculation, however, this particular combination did not arise naturally. Moreover, division by s^2 can give rise to an important enhancement, and such an enhancement might well have affected our results qualitatively.

It is interesting to note that there exists a small negative contribution to T', due to the additional V piece in $T' = T + V = -0.02$. Thus, we find that even a perfectly degenerate triplet of scalars yields a non-zero (and negative!) contribution to the effective ρ parameter measured at the Z-pole. This result is not without phenomenological pertinence: for example, a 10 GeV deviation of the top mass from a fiducial value of 178 GeV gives a change in T of the same order, namely $\approx \pm 0.06$.

4. Conclusion

The 'S-argument' against Technicolor hinges on the fact that the value of S calculated in a one-family Technicolor model is large and positive, while the experimental measurements of S at the Z-pole are consistent with zero. In a one-family Technicolor model with light pseudo-Goldstone bosons the parameter that is measured at the Z-pole is S',

where $S' \equiv S + 4(c^2 - s^2)X + 4c^2s^2V$. Thus it is clear that if either X or V are large and negative the calculated value of S' can be consistent with the experimental data. The result of such a calculation can not be guessed *a priori*. We have calculated the parameters $STUVWX$ in a one-family Technicolor model with light pseudo-Goldstone bosons, and found that the values of V, and X do not contribute significantly to S'. Hence one-family Technicolor models with light psuedo-Goldstone bosons can not counter the 'S-argument' against Technicolor.

Though the values of V, and X do not play a predominant role. One ought to keep in mind though that, as is discussed in refs. [2], [7] and [8], there do indeed exist models of new physics in which the extended set of parameters may well be relevant. Thus, it is possible that the $STUVWX$ parameter set might one day participate in untangling some signal of physics beyond the Standard Model.

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